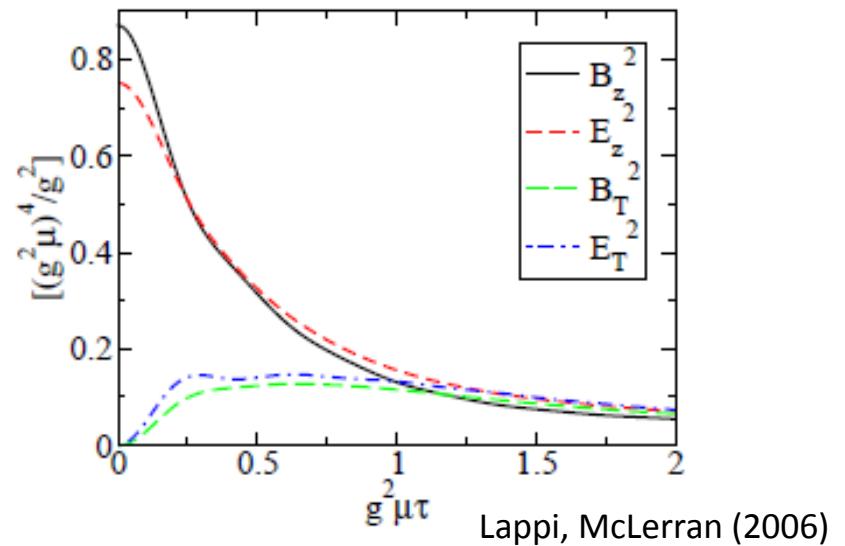
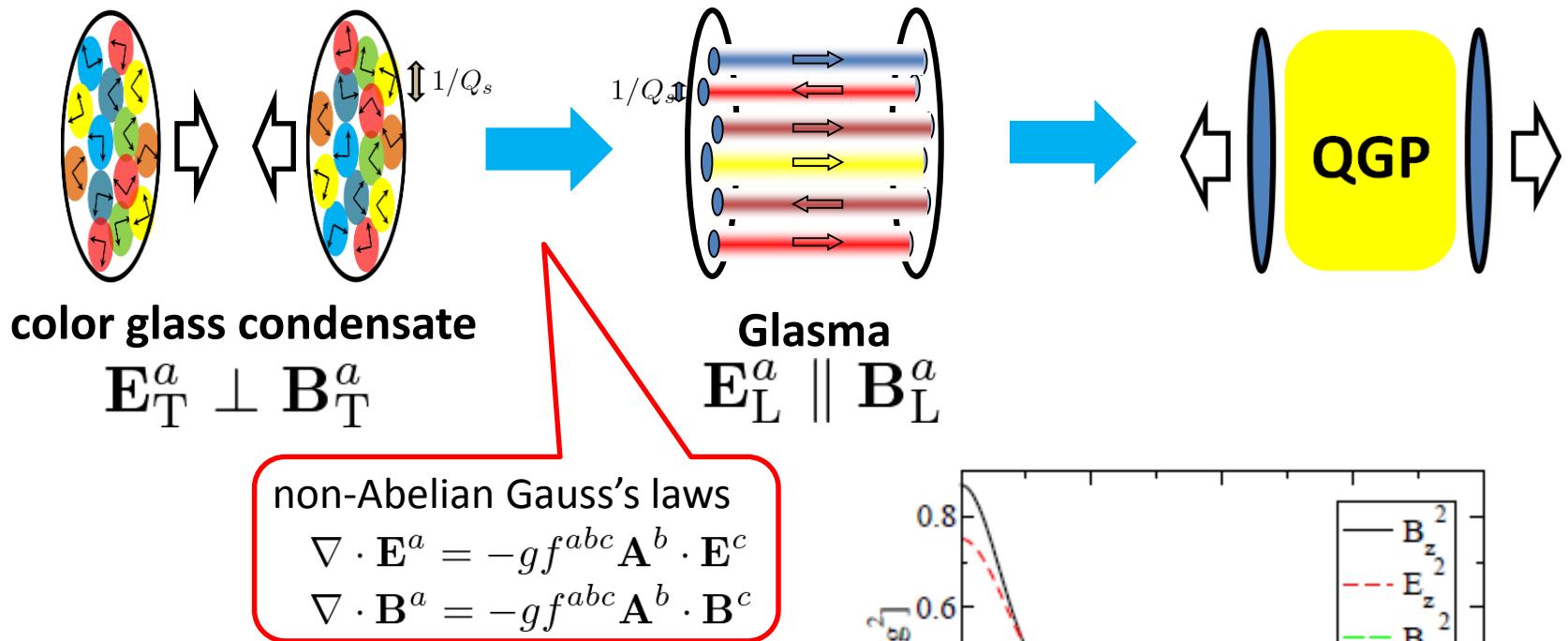


Quark Production in Glasma

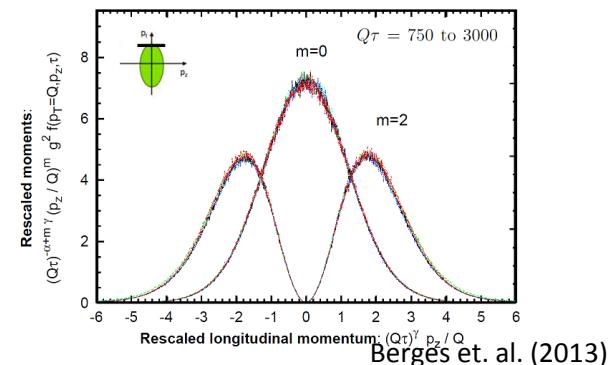
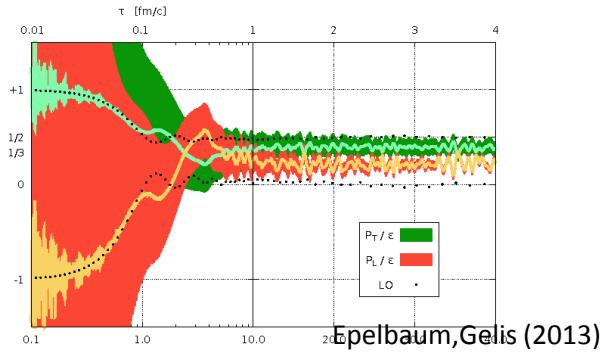
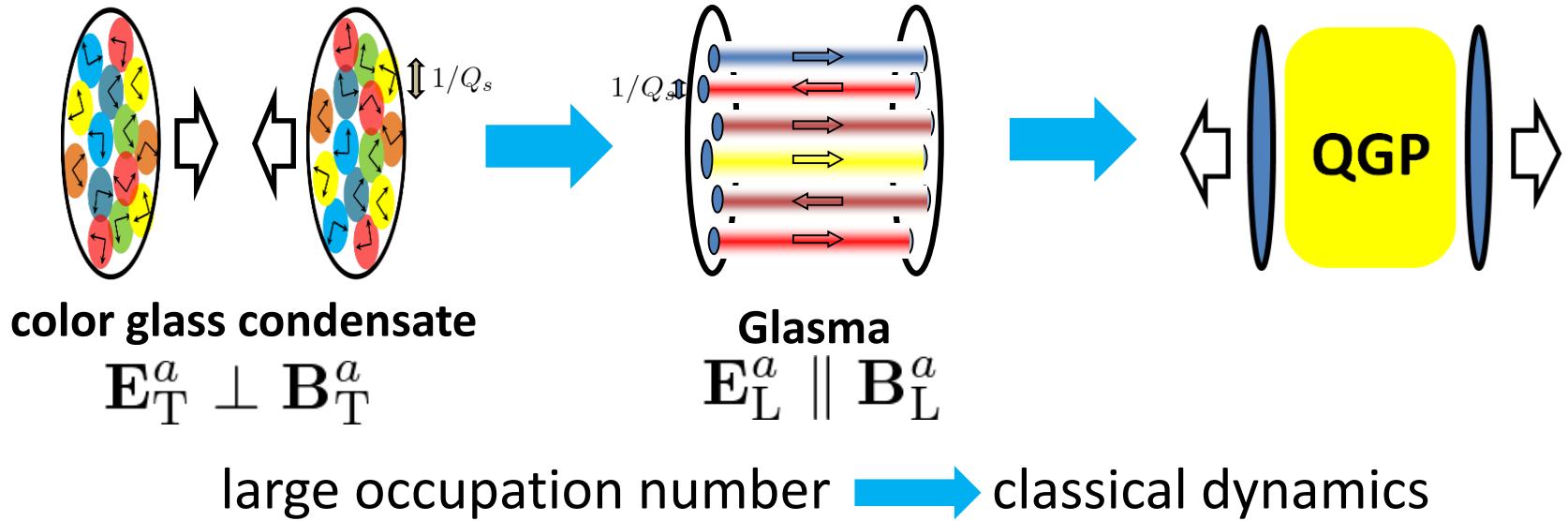
Naoto Tanji
RIKEN (-> BNL in May 2014)
collaboration with F. Gelis (Saclay)

Strong fields in heavy-ion collisions



Strong fields in heavy-ion collisions

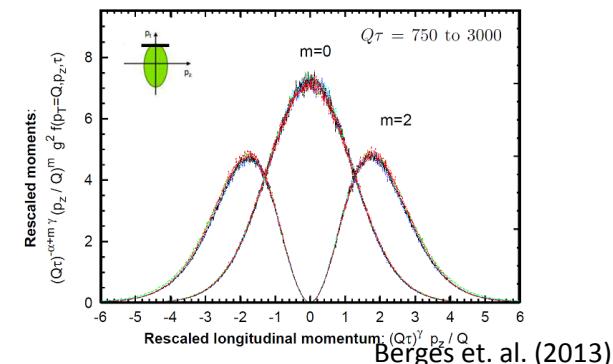
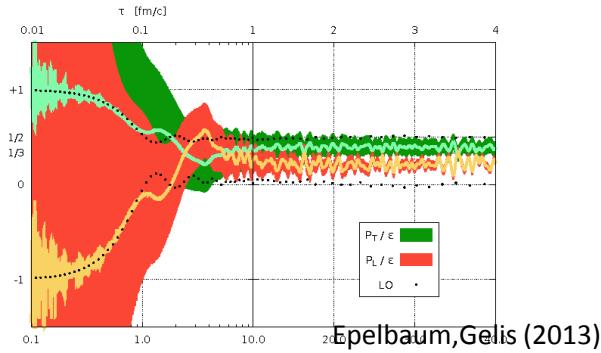
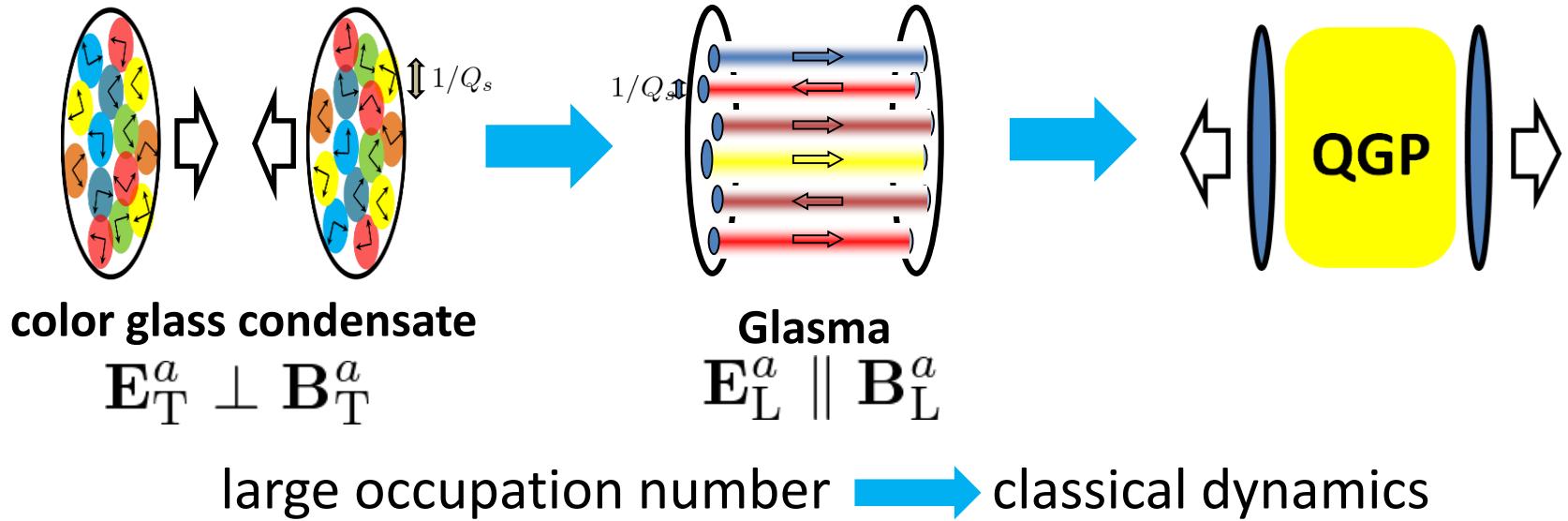
The time-evolution of Glasma toward isotropization and thermalization



Classical statistical simulations of gluodynamics

Strong fields in heavy-ion collisions

The time-evolution of Glasma toward isotropization and thermalization



Classical statistical simulations of gluodynamics
How about quarks?

Quark production

CGC, plasma



purely gluonic matter

How does the system reaches chemical equilibrium
between light quarks and gluons?

Earlier works by F. Gelis, K. Kajantie and T. Lappi

PRC71, 024904(2005)

PRL96, 032304(2006)

- Limitation from numerical costs
- Treatment of the boost-invariance

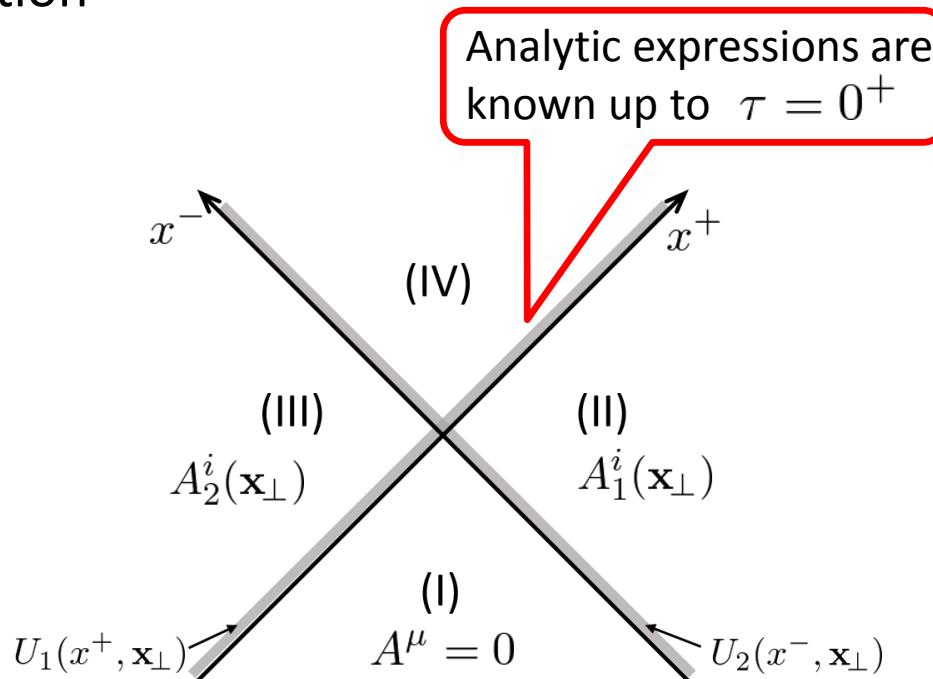
There have been theoretical and technical advances on

- classical statistical method for over-occupied bosonic fields
- real-time lattice simulations of fermionic fields
- treatment of a boost-invariant system

Setup

□ gauge fields

- SU(2)
- boost-invariant classical fields without fluctuations
- generated by colliding CGCs with the McLerran-Venugopalan model
- in the forward light cone, solve the classical YM eq. numerically
- no back reaction



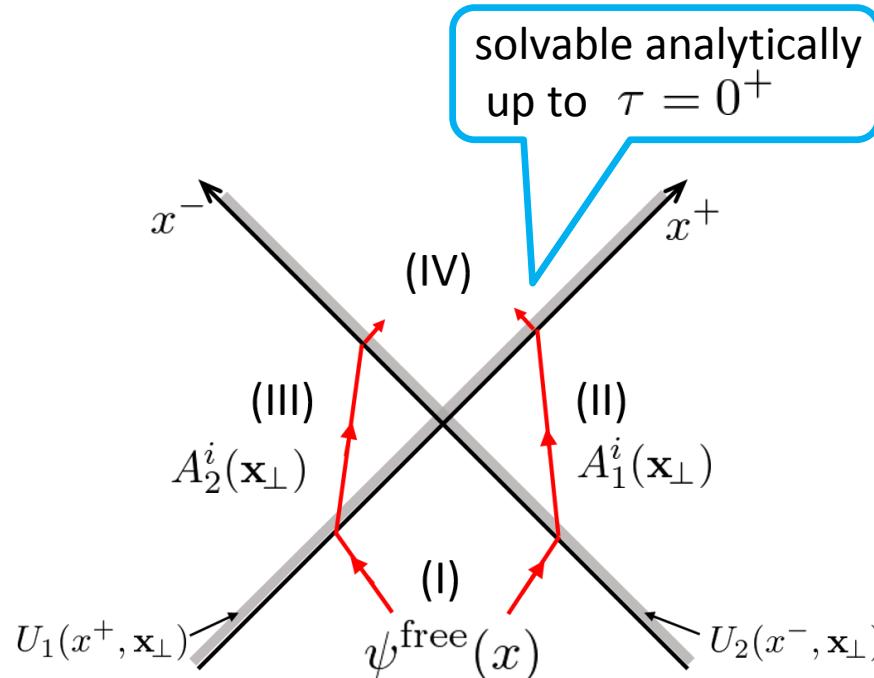
Setup

□ quark fields

- obey the Dirac equation

$$[i\gamma^\mu(\partial_\mu + igA_\mu) - m] \hat{\psi}(x) = 0$$

- free fields in the region (I)
- in the forward light cone, solve the Dirac eq. numerically



Solving the Dirac equation numerically

- Mode functions approach Aarts, Smit (1998)

$$[i\gamma^\mu(\partial_\mu + igA_\mu) - m] \hat{\psi}(x) = 0 \quad \text{linear in the Dirac field}$$

$$\hat{\psi}(x) = \sum_s \int d^3p [\psi_{\mathbf{p},s}^+(x) a_{\mathbf{p},s} + \psi_{\mathbf{p},s}^-(x) b_{\mathbf{p},s}^\dagger]$$

$\psi_{\mathbf{p},s}^\pm(x)$: mode functions, c-number solutions of the Dirac eq.

$$[i\gamma^\mu(\partial_\mu + igA_\mu) - m] \psi_{\mathbf{p},s}^\pm(x) = 0 \quad \lim_{t \rightarrow -\infty} \psi_{\mathbf{p},s}^\pm(x) = \psi_{\mathbf{p},s}^{\text{free},\pm}(x)$$

$$J_{\text{quark}}^\mu = \frac{g}{2} \langle 0 | [\hat{\bar{\psi}}(x), \gamma^\mu \hat{\psi}(x)] | 0 \rangle$$

$$= -\frac{g}{2} \sum_s \int d^3p \{ \bar{\psi}_{\mathbf{p},s}^+(x) \gamma^\mu \psi_{\mathbf{p},s}^+(x) - \bar{\psi}_{\mathbf{p},s}^-(x) \gamma^\mu \psi_{\mathbf{p},s}^-(x) \}$$

With the c-number mode functions, expectation values can be computed.

Solving the Dirac equation numerically

- Mode functions approach Aarts, Smit (1998)

$$[i\gamma^\mu(\partial_\mu + igA_\mu) - m] \hat{\psi}(x) = 0 \quad \text{linear in the Dirac field}$$

$$\hat{\psi}(x) = \sum_s \int d^3p [\psi_{\mathbf{p},s}^+(x) a_{\mathbf{p},s} + \psi_{\mathbf{p},s}^-(x) b_{\mathbf{p},s}^\dagger]$$

$\psi_{\mathbf{p},s}^\pm(x)$: mode functions, c-number solutions of the Dirac eq.

$$[i\gamma^\mu(\partial_\mu + igA_\mu) - m] \psi_{\mathbf{p},s}^\pm(x) = 0 \quad \lim_{t \rightarrow -\infty} \psi_{\mathbf{p},s}^\pm(x) = \psi_{\mathbf{p},s}^{\text{free},\pm}(x)$$

$$\begin{aligned} J_{\text{quark}}^\mu &= \frac{g}{2} \langle 0 | [\hat{\bar{\psi}}(x), \gamma^\mu \hat{\psi}(x)] \rangle \\ &= -\frac{g}{2} \sum_s \int d^3p \{ \bar{\psi}_{\mathbf{p},s}^+(x) \psi_{\mathbf{p},s}^-(x) \} \end{aligned}$$

Numerical cost

$$N_t \times N_{\text{latt}}^2$$

expensive in 3+1dim.

With the c-number mode functions, expectation values can be computed.

□ Monte Carlo method with male and female stochastic fields

Borsanyi, Hindmarsh (2009)

➤ Male and female fields

- initial condition

$$\psi_M(t_0, \mathbf{x}) = \sum_s \int d^3p [\psi_{\mathbf{p},s}^+(t_0, \mathbf{x}) c_{\mathbf{p},s} + \psi_{\mathbf{p},s}^-(t_0, \mathbf{x}) d_{\mathbf{p},s}]$$

$$\psi_F(t_0, \mathbf{x}) = \sum_s \int d^3p [\psi_{\mathbf{p},s}^+(t_0, \mathbf{x}) c_{\mathbf{p},s} - \psi_{\mathbf{p},s}^-(t_0, \mathbf{x}) d_{\mathbf{p},s}]$$

- evolution equation

$$[i\gamma^\mu(\partial_\mu + igA_\mu) - m] \psi_{M,F}(x) = 0$$

➤ Gaussian random numbers

$$\langle c_{\mathbf{p},s} c_{\mathbf{p}',s'}^* \rangle = \langle d_{\mathbf{p},s} d_{\mathbf{p}',s'}^* \rangle = \frac{1}{2} \delta_{s,s'} \delta^3(\mathbf{p} - \mathbf{p}')$$

Expectation values can be reproduced by male-female ensemble average.

$$\begin{aligned} -g \langle \bar{\psi}_M(x) \gamma^\mu \psi_F \rangle &= -\frac{g}{2} \sum_s \int d^3p \{ \bar{\psi}_{\mathbf{p},s}^+(x) \gamma^\mu \psi_{\mathbf{p},s}^+(x) - \bar{\psi}_{\mathbf{p},s}^-(x) \gamma^\mu \psi_{\mathbf{p},s}^-(x) \} \\ &= J_{\text{quark}}^\mu \end{aligned}$$

□ Monte Carlo method with male and female stochastic fields

Borsanyi, Hindmarsh (2009)

➤ Male and female fields

- initial condition

$$\psi_M(t_0, \mathbf{x}) = \sum_s \int d^3p [\psi_{\mathbf{p},s}^+(t_0, \mathbf{x}) c_{\mathbf{p},s} + \psi_{\mathbf{p},s}^-(t_0, \mathbf{x}) d_{\mathbf{p},s}]$$

$$\psi_F(t_0, \mathbf{x}) = \sum_s \int d^3p [\psi_{\mathbf{p},s}^+(t_0, \mathbf{x}) c_{\mathbf{p},s} - \psi_{\mathbf{p},s}^-(t_0, \mathbf{x}) d_{\mathbf{p},s}]$$

- evolution equation

$$[i\gamma^\mu(\partial_\mu + igA_\mu) - m] \psi_{M,F}(x) = 0$$

➤ Gaussian random numbers

$$\langle c_{\mathbf{p},s} c_{\mathbf{p}',s'}^* \rangle = \langle d_{\mathbf{p},s} d_{\mathbf{p}',s'}^* \rangle = \frac{1}{2} \delta_{s,s'} \delta^3(\mathbf{p} - \mathbf{p}')$$

Expectation values can be reproduced by male-female ensemble average.

$$\begin{aligned} -g \langle \bar{\psi}_M(x) \gamma^\mu \psi_F \rangle &= -\frac{g}{2} \sum_s \int d^3p \{ \bar{\psi}_{\mathbf{p},s}^+(x) \gamma^\mu \psi_{\mathbf{p},s}^+(x) - \bar{\psi}_{\mathbf{p},s}^-(x) \gamma^\mu \psi_{\mathbf{p},s}^-(x) \} \\ &= J_{\text{quark}}^\mu \end{aligned}$$

The male-female combination is necessary to get this minus sign which originates in anti-commutativity.

□ Monte Carlo method with male and female stochastic fields

Borsanyi, Hindmarsh (2009)

➤ Male and female fields

- initial condition

$$\psi_M(t_0, \mathbf{x}) = \sum_s \int d^3p [\psi_{\mathbf{p},s}^+(t_0, \mathbf{x}) c_{\mathbf{p},s} + \psi_{\mathbf{p},s}^-(t_0, \mathbf{x}) d_{\mathbf{p},s}]$$

$$\psi_F(t_0, \mathbf{x}) = \sum_s \int d^3p [\psi_{\mathbf{p},s}^+(t_0, \mathbf{x}) c_{\mathbf{p},s} - \psi_{\mathbf{p},s}^-(t_0, \mathbf{x}) d_{\mathbf{p},s}]$$

Numerical cost

$$N_{\text{config}} \times N_{\text{latt}}^2$$

- evolution equation

$$[i\gamma^\mu(\partial_\mu + igA_\mu) - m] \psi_{M,F}(x) = 0$$

➤ Gaussian random numbers

$$\langle c_{\mathbf{p},s} c_{\mathbf{p}',s'}^* \rangle = \langle d_{\mathbf{p},s} d_{\mathbf{p}',s'}^* \rangle = \frac{1}{2} \delta_{s,s'} \delta^3(\mathbf{p} - \mathbf{p}')$$

Expectation values can be reproduced by male-female ensemble average.

Total cost

$$-g\langle \bar{\psi}_M \psi_M + \bar{\psi}_F \psi_F \rangle = N_{\text{config}} \times N_{\text{latt}} \times (N_{\text{latt}} + N_t) \ll N_t \times N_{\text{latt}}^2$$

if $N_{\text{config}} \ll N_{\text{latt}}$ and $N_{\text{config}} \ll N_t$

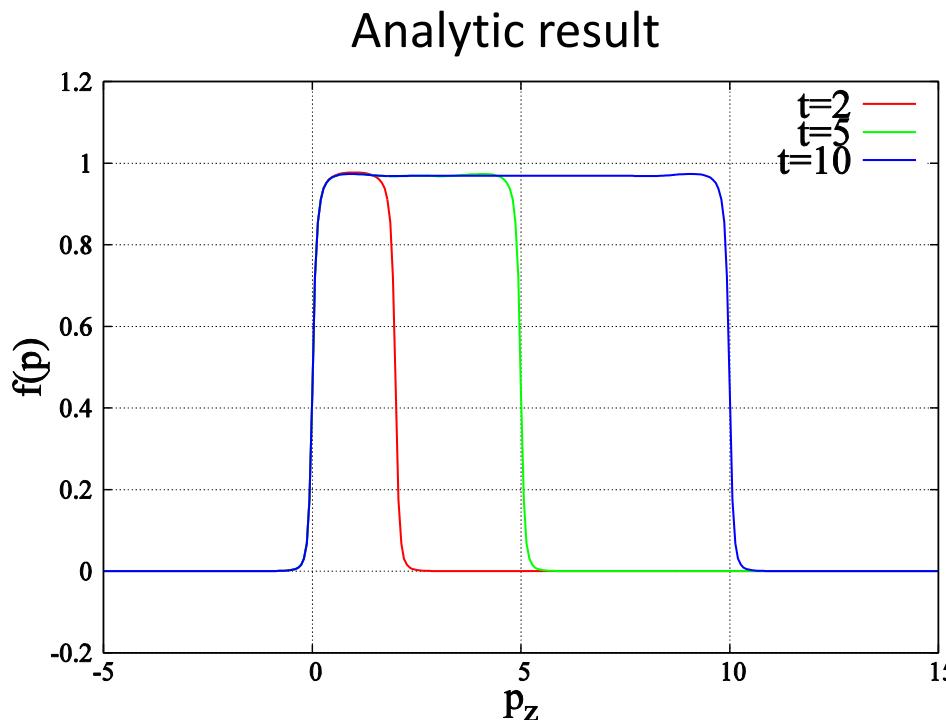
It is necessary
to get this minus sign which originates in
anti-commutativity.

Benchmark --- QED uniform and constant electric field

Schwinger mechanism particle pair production

NT, Ann.Phys.324(2009)

$$eE = 1$$
$$m = 0.1$$

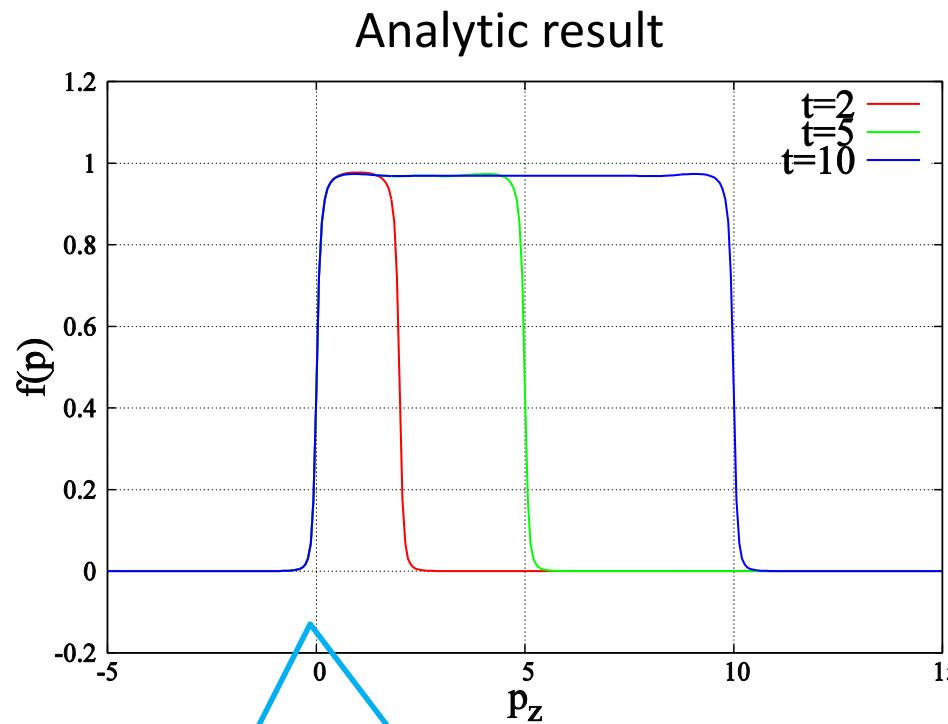


Benchmark --- QED uniform and constant electric field

Schwinger mechanism particle pair production

NT, Ann.Phys.324(2009)

$$eE = 1$$
$$m = 0.1$$



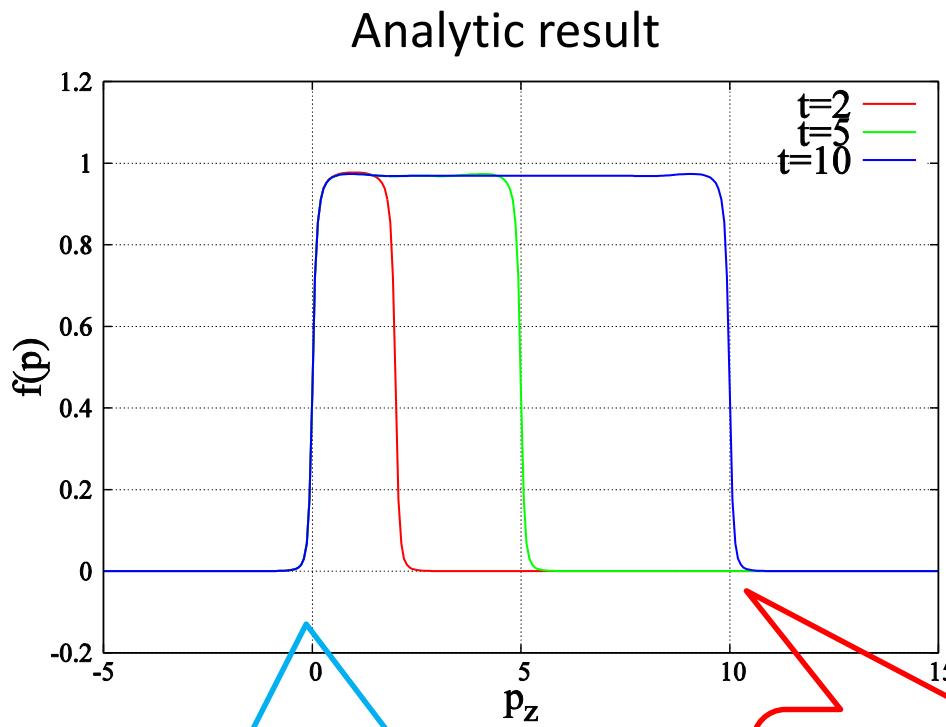
created with approximately
0 longitudinal momentum

Benchmark --- QED uniform and constant electric field

Schwinger mechanism particle pair production

NT, Ann.Phys.324(2009)

$$eE = 1$$
$$m = 0.1$$



created with approximately
0 longitudinal momentum

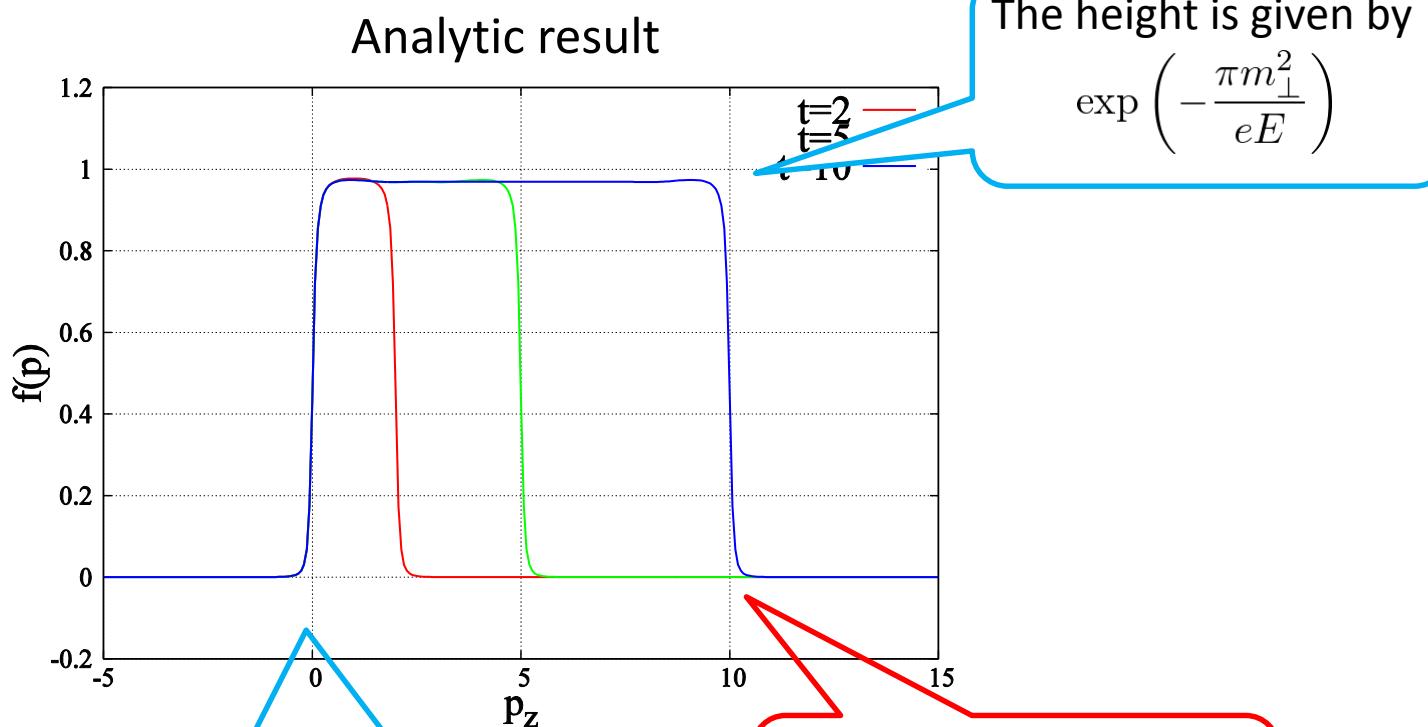
accelerated according to
classical eq. of motion
 $p_z = eEt$

Benchmark --- QED uniform and constant electric field

Schwinger mechanism particle pair production

NT, Ann.Phys.324(2009)

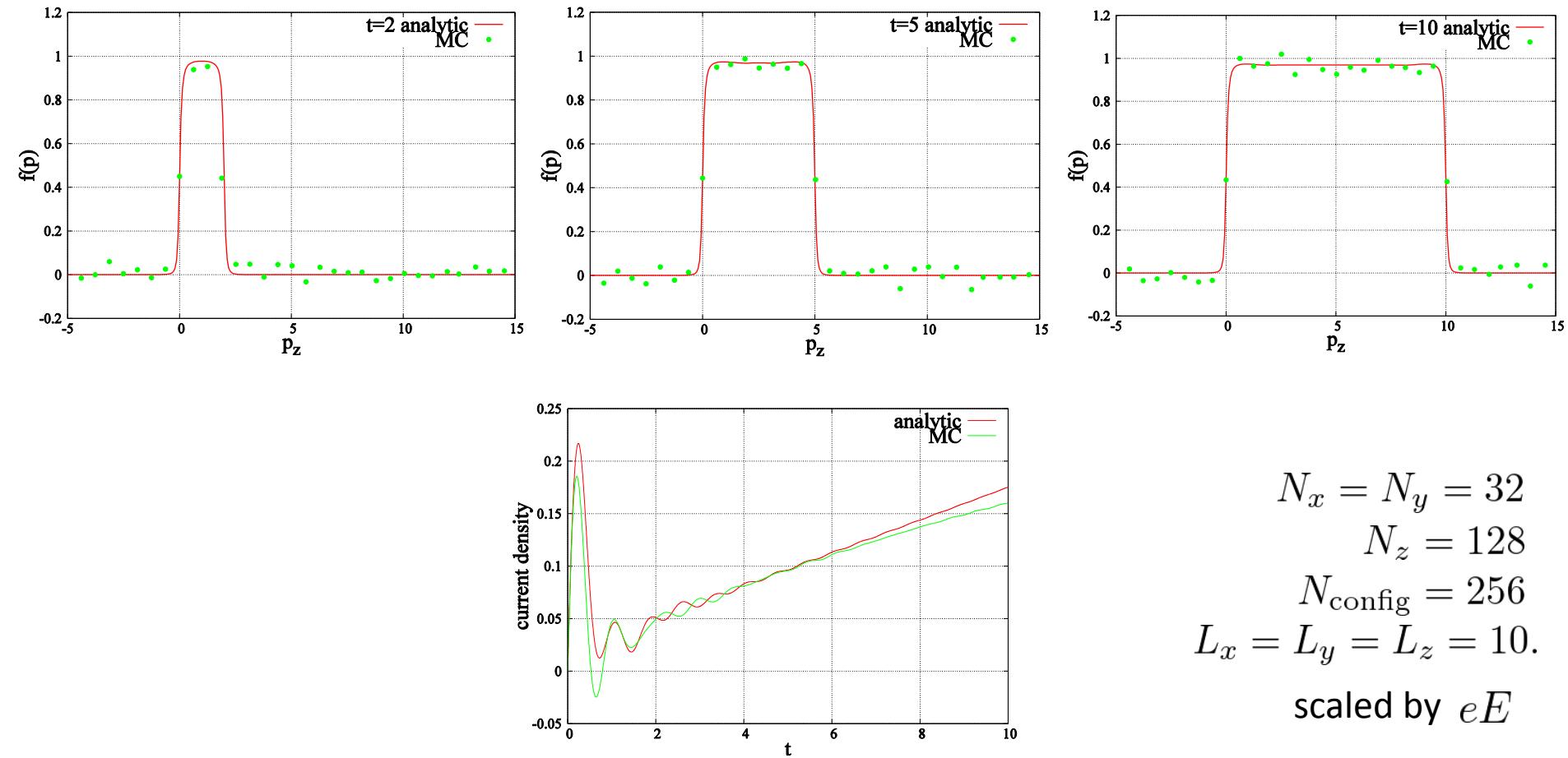
$$eE = 1$$
$$m = 0.1$$



The height is given by
$$\exp\left(-\frac{\pi m_\perp^2}{eE}\right)$$

Benchmark --- QED uniform and constant electric field

Comparison between the analytic and MC results



$$\begin{aligned}N_x &= N_y = 32 \\N_z &= 128 \\N_{\text{config}} &= 256 \\L_x &= L_y = L_z = 10.\end{aligned}$$

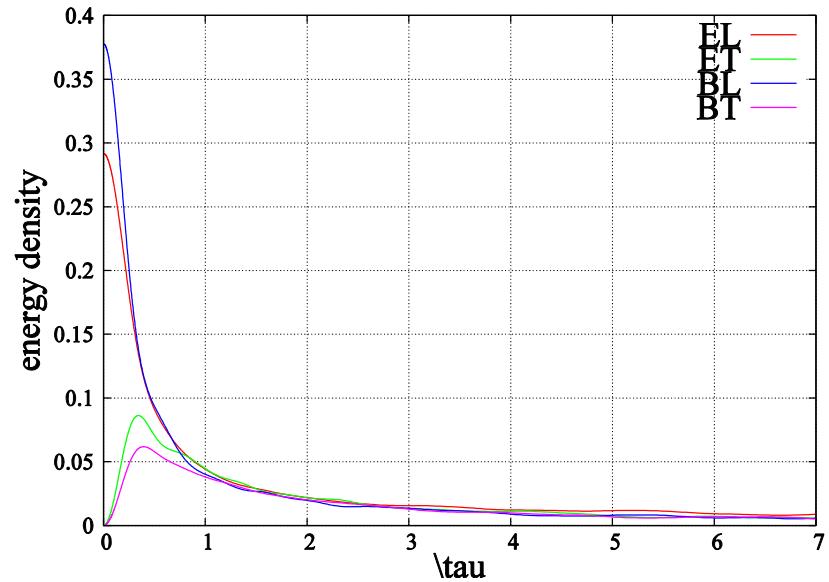
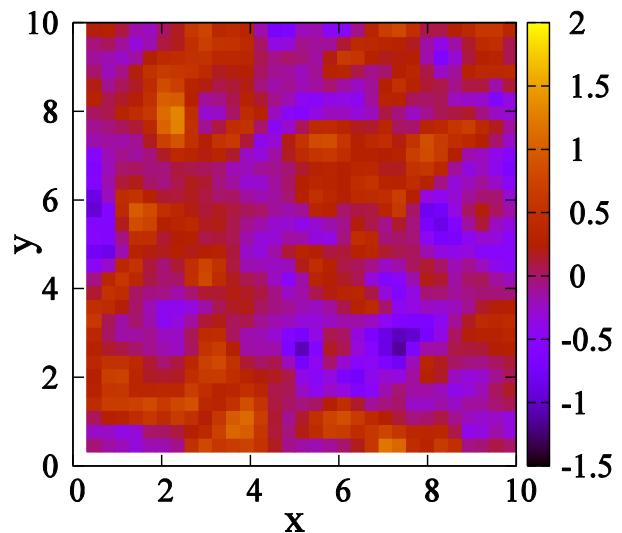
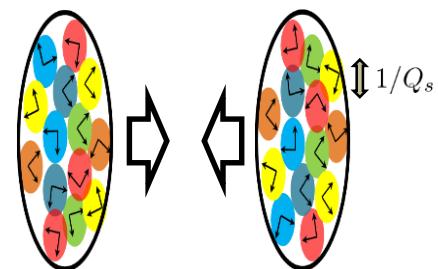
scaled by eE

The MC method well reproduces the analytic results.

McLerran-Venugopalan initial condition

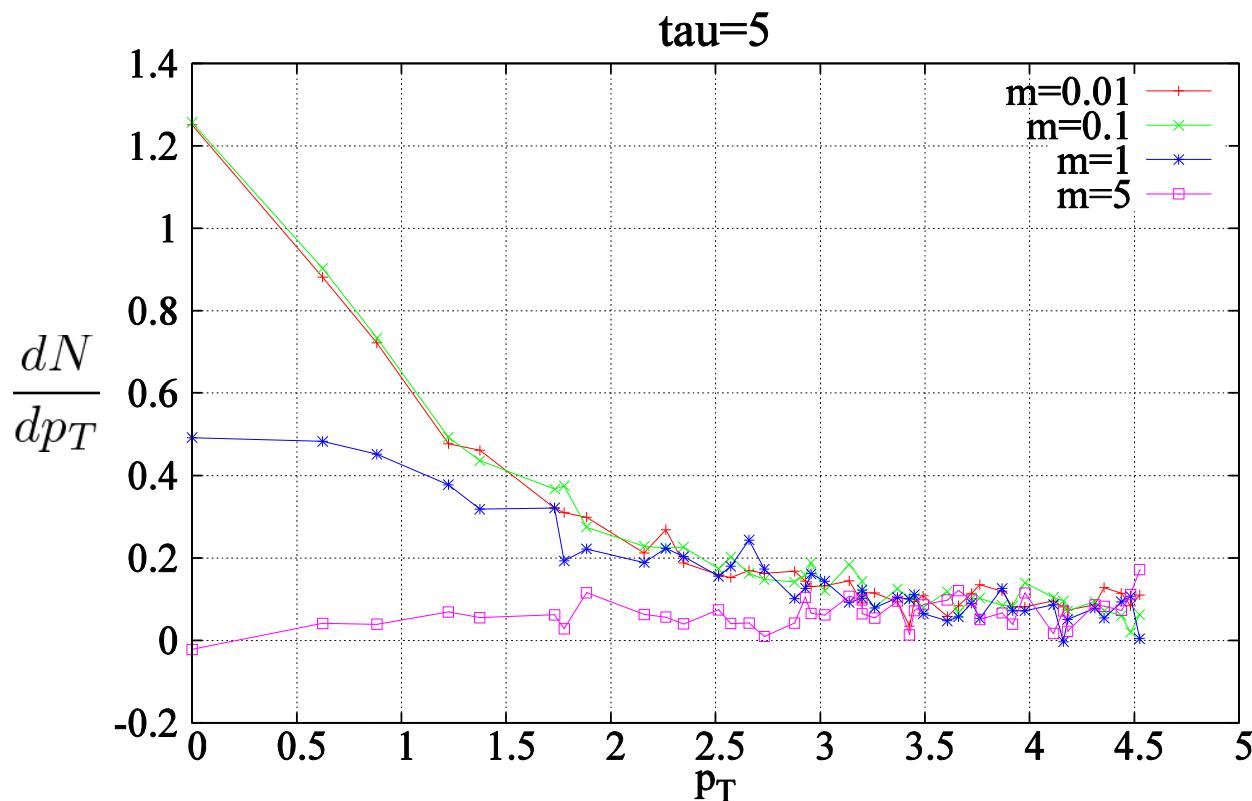
Random color sources

$$\langle \rho^{(n)}(x_\perp) \rho^{(m)}(x'_\perp) \rangle = g^2 \mu^2 \delta^{nm} \delta(x_\perp - x'_\perp)$$



All dimensionful quantities are scaled by $g\mu$.

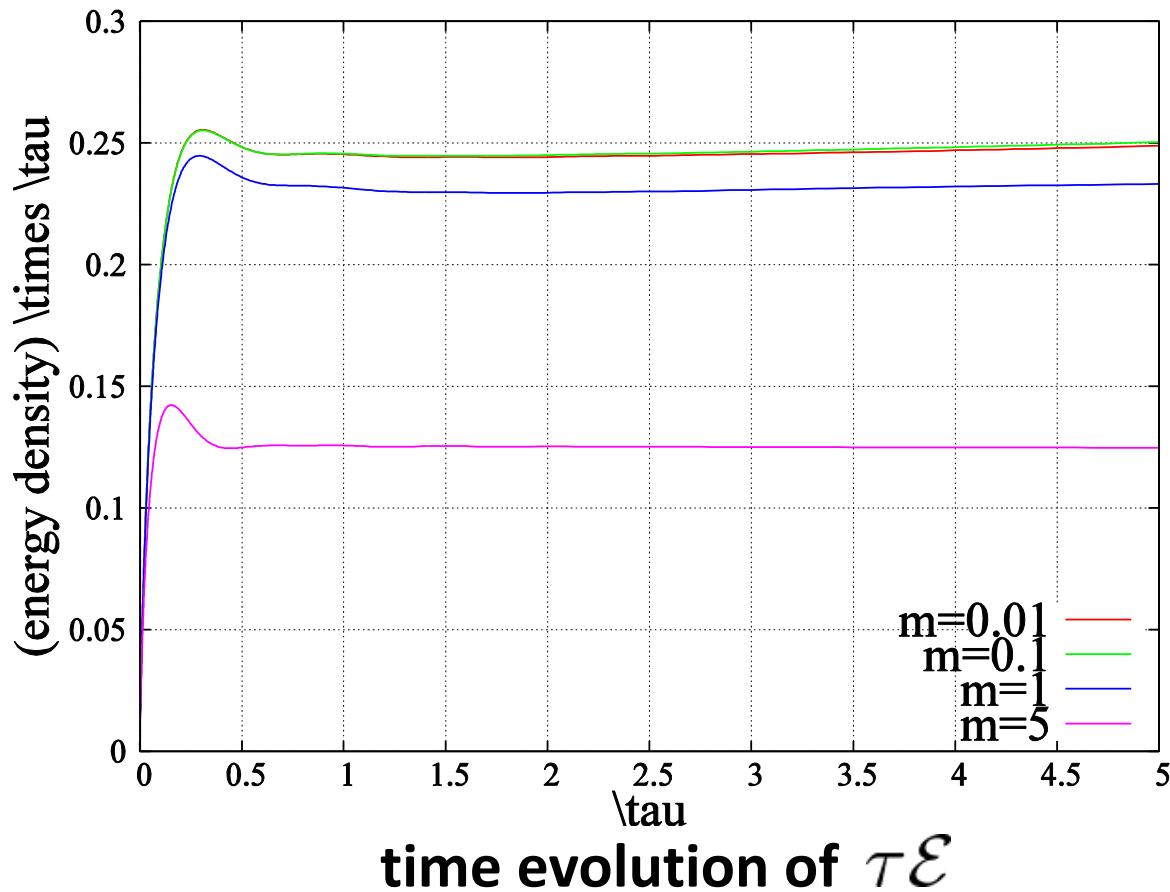
Transverse spectra of produced particles



$N_x = N_y = 32$
 $N_\eta = 128$
 $N_{\text{config}} = 256$
 $L_x = L_y = 10.$
 $L_\eta = 20.$
scaled by $g\mu$

mass dependence of the transverse spectra

Energy density



$$N_x = N_y = 32$$

$$N_\eta = 128$$

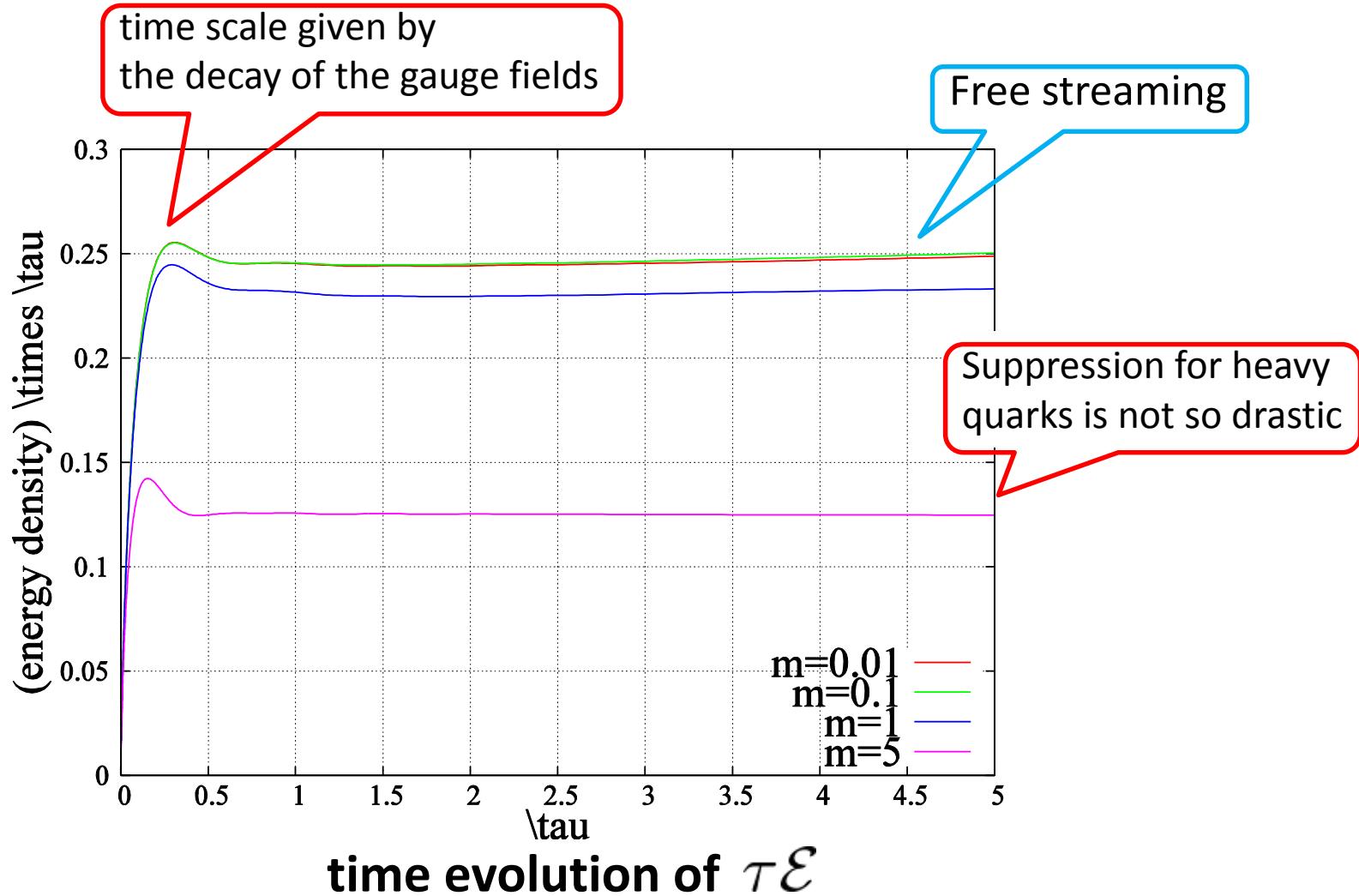
$$N_{\text{config}} = 256$$

$$L_x = L_y = 10.$$

$$L_\eta = 20.$$

scaled by $g\mu$

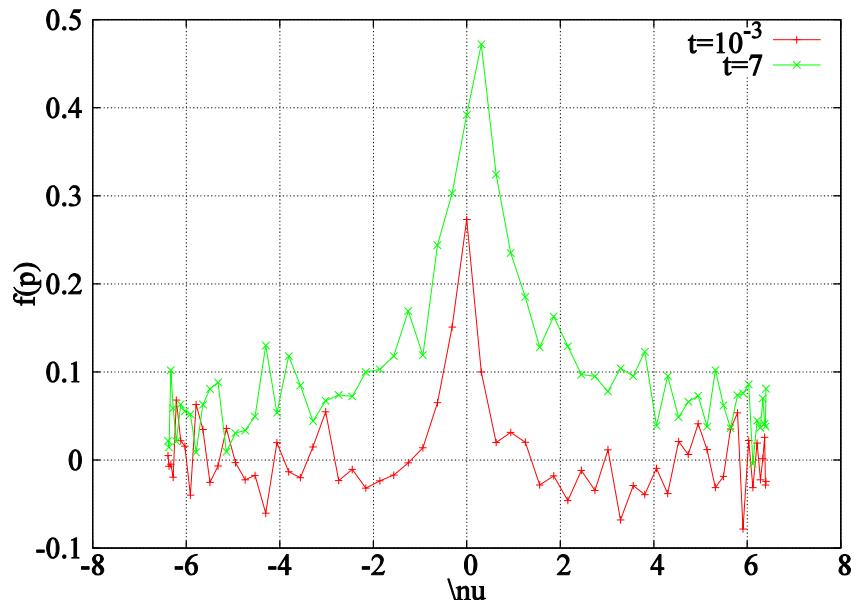
Energy density



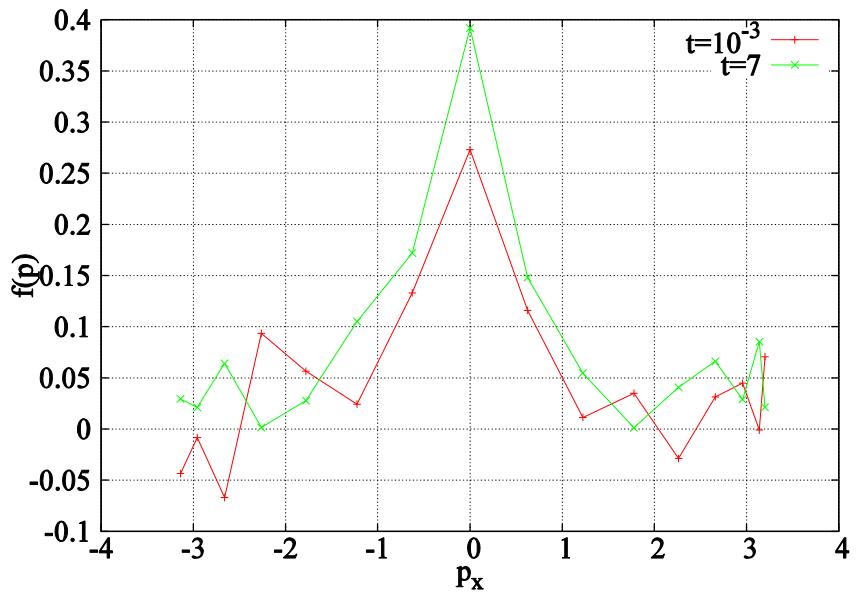
Summary

- The MC method enables us to simulate fermion dynamics in inhomogeneous classical gauge fields.
- The quark production in expanding gauge fields with the MV initial condition is computed.
- Prompt increase of energy density of produced quarks is obtained.

Spectra of produced particles



ν -dependence of the distribution
with fixed transverse momentum $p_T = 0$



p_x -dependence of the distribution
with $\nu = p_y = 0$

ν : momentum conjugate to space-time rapidity η

$p_z = \nu/\tau$: momentum observed in a frame moving with the velocity $v_z = z/t = \tanh \eta$

Mass dependence for the Schwinger mechanism

